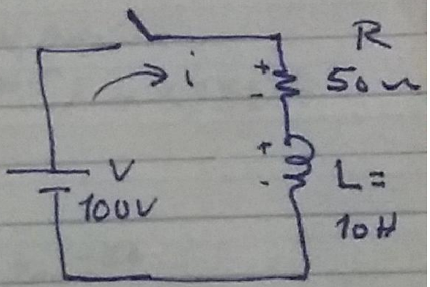
sheet (7)

1) A series RL with $R = 50\,\Omega$, and $L = 10\,H$, has a constant voltage $V = 100\,V$ applied at $T = 0$ by closing of a switch. Find
(a) equations for i, V_R, and V_L.
(b) The current at $t = 0.5\,s$
(c) The time at which $V_R = V_L$
(d) Find the equations for P_R and P_L and show that the power in inductance accounts for the steady state energy in the magnetic field.

⟵ Solution ⟶



By closing switch $\Sigma V = 0$

$$+100 - (i)(50) - L\frac{di}{dt} = 0$$

$$100 = 50i + 10\,di/dt$$

$$100 = i(50 + 10D) = 10i(5+D)$$

$$\therefore \boxed{10 = i(5+D)} \rightarrow 2\ \text{solutions} \rightarrow \text{P.I},\ \text{Complementry}$$

for P.I ~~complementry~~ sol. Put $D = 0$ $\therefore i = 10/5 = 2$

for complementry sol.

تفاضل الطرفين

$$(m+5)i = 10$$

$$0 = m + 5 \qquad m = -5$$

$$\therefore sol_c = Ae^{mt} = Ae^{-5t}$$

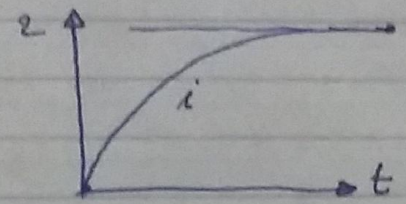
$$i_t = Ae^{-5t} + 2$$

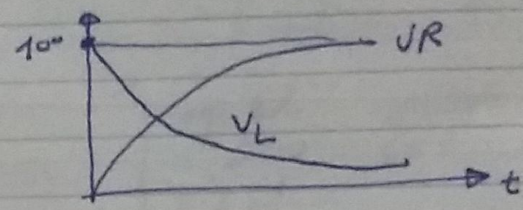
at $t=0 \rightarrow i_0 = 0$ $\therefore$ $0 = 2 + A e^{-0}$ $\therefore$ $A = -2$

$$\therefore \boxed{i_t = -2e^{-5t} + 2 = 2(1 - e^{-5t})}$$

$\rightarrow V_R = iR = 100(1 - e^{-5t})$ ※

$\rightarrow V_L = L\frac{di}{dt} = 10 * [-2e^{-5t} \times -5] = 100 e^{-5t}$ ※





(b) current at $t = 0.5$ نعوض في المعادلة مباشرة

$i = 2(1 - e^{-5t}) = 2(1 - e^{-5\times0.5}) = 1.836\,A$

(c) when $V_R = V_L$ find t

$100(1 - e^{-5t}) = 100 e^{-5t}$ $\therefore$ $100 = 200 e^{-5t}$

$\therefore e^{-5t} = \frac{1}{2}$ $\ln e^{-5t} = \ln \frac{1}{2}$

$\therefore -5t = \ln\frac{1}{2} / \ln e$ or $t = \frac{-1}{5} \ln\frac{1}{2} / \ln e$

$t = 0.1386$ sec

(d) P_R, P_L $* P_R = V_R i = 100(1 - e^{-5t})(2(1 - e^{-5t}))$

$= 200(1 - 2e^{-5t} + e^{-10t})$ ※

$P_L = V_L i = (100 e^{-5t})(2(1 - e^{-5t})) = 200(e^{-5t} - e^{-10t})$

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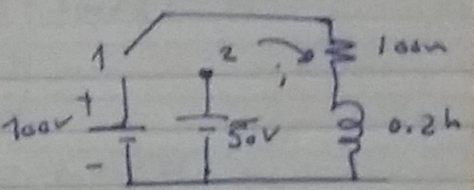
* ~~steady~~

ملحوظة [illegible] $i_0 = i_0$, [illegible] $X_L = 2\pi f L = 2\pi \frac{1}{t} L = \frac{1}{0} = \infty$ $\therefore i' = 0$

2 In The Series Circuit shown in fig. 1, the Switch is closed on Position 1 at $t=0$, Thereby applying The 100 Volt Source to The RL Branch, and at $t = 500\,\mu sec$ the Switch is moved to Position 2 obtain The equations for current in both Intervals and Sketch Transient.

Sol.



→ at Pos. 1

$$+100 = 100 i + 0.2 \frac{di}{dt}$$

$$500 = 500 i + di/dt$$

$$(D + 500) i = 500$$

$$\boxed{i_{t_1} = 1 + A e^{-500t}}$$

Comp. sol. $D=0$ $i=1$

P.I sol. $0 = m + 500 \rightarrow m = -500$

$\therefore A e^{-500t} + 1 = i_t$

at $t=0$ $i_0 = 0$ $A = -1$

$$\therefore \boxed{i_{t_1} = 1 - e^{-500t}} \longrightarrow \text{I} \quad \text{for} \quad 0 < t < t_1$$

→ at $t = 500\,\mu sec$ $\quad i = 1 - e^{-500 \times 500\mu} = 0.221\,Amp$

Switch Now at position 2, $V_{source} = 50V$

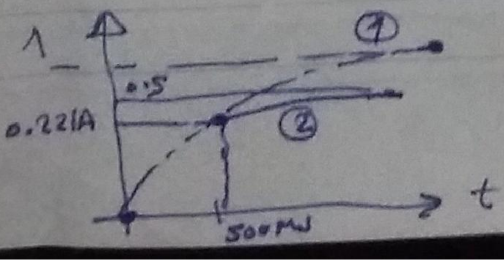
$\therefore \quad 50 = 100 i + 0.2\, di/dt \qquad$ or $(D + 500) = 250$

$i_{t_2} = 0.5 + B e^{-500 t_2} \qquad$ where $t_2 = t - \underbrace{t_1}_{500\mu s}$

at $t = t_1$. $t_2 = 0$, $i_2 = 0.221A$

$\therefore \quad 0.22A = 0.5 + B \quad \therefore \boxed{B = -0.279}$

for $t > t_1$ $\therefore$ $\boxed{i_{t2} = 0.5 - 0.279\, e^{-500(t - \overset{500\mu s}{t_1})}}$ → for $t > t_1$



3 Repeat 2 with 50 volt polarity Reversed.

$i_{t_1} = 1 - e^{-500t} \longrightarrow \mathrm{I}$ كما هو مبين

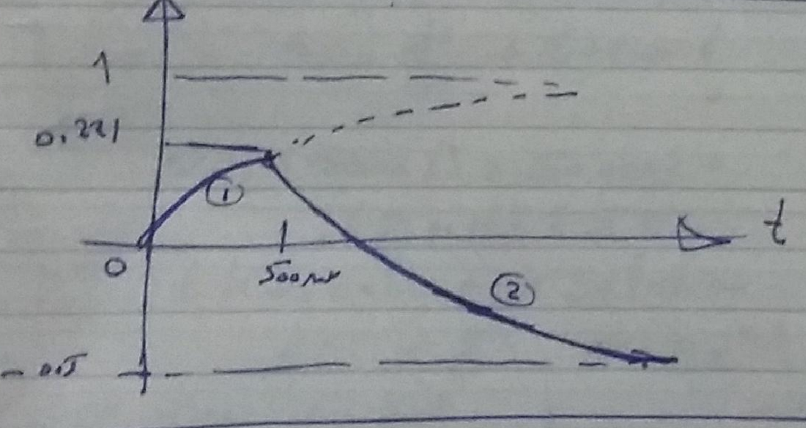
i at 500 μsec = 0.221 A كما هو

Now $100 i + 0.2\, di/dt = -50$ $(D + 500)\, i = -250$

$\therefore \boxed{i_2 = B_2 e^{-500 t_2} - 0.5}$

for $t = t_1 \Rightarrow i_2 = 0.22$ $\therefore B_2 = +0.721$

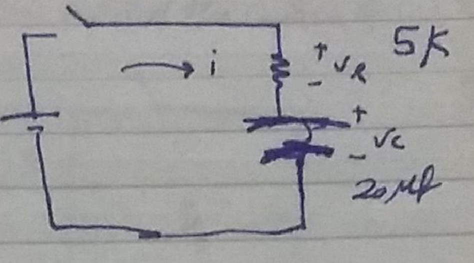
$$\therefore \boxed{i_{2t} = +0.721\, e^{-500(t - t_1)} - 0.5}$$



[4] A series RC circuit with $R = 5000\,\Omega$ and $C = 20\,\mu f$ has a constant voltage $V = 100\,v$ applied at $t = 0$ and the capacitor has no initial charge. Find the equations of i, V_R, and V_C.

<u>Sol.</u>

<u>S closed</u>



$$100 = i(5000) + \frac{1}{C}\int i\,dt \rightarrow \boxed{1}$$

$$100 = 5000\,i + \frac{1}{20\times10^{-6}}\int i\,dt$$

$$100 = 5000\,i + 50000\int i\,dt$$

تفاضل الطرفين للتخلص من التكامل

$$0 = 5000\frac{di}{dt} + 50000\,i$$

$$0 = (5000D + 50000)\,i$$

$$(D+10)\,i = 0$$

it has only one solution (P.I) $\quad m+10=0$, $\; m=-10$

$$i = Ae^{mt} = Ae^{-10t}$$

at $t=0$ → sub in 1 $\quad \therefore\; 100 = 5000\,i\;$ or $\;i = 100/5000$

$$i = 0.02\,A$$

$$\therefore\; \boxed{i = 0.02\,e^{-10t}}$$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi C}\cdot\frac{1}{0} = \infty$ ∴ [illegible] $= V/R$

$$V_R = Ri = 5000\times0.02\,e^{-10t} = 100\,e^{-10t}$$

$$V_C = \frac{1}{C}\int i\,dt = \frac{1}{20\times10^{-6}}\int 0.02\,e^{-10t}\,dt = 100(1-e^{-10t})$$

$$V_C = -100\,e^{-10t} + C$$

at $t=0$ $\quad V_R = 100,\; V = 100 \;\therefore V_C = 0$

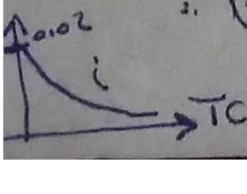
$$\therefore\; 0 = -100\,e^{0} + C \;,\; C = 100$$

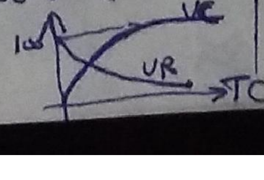
$$\therefore\; V_C = 100 - 100\,e^{-10t} = 100(1-e^{-10t})$$

$\left[\int e^{-10t}\,dt\right]1000$

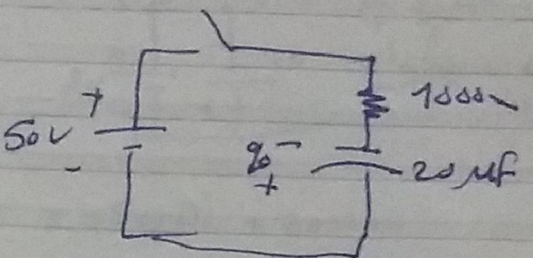
$$1000\,\frac{e^{-10t}}{-10} + C = V_C$$

at $t=0$, $V_C = 0$ $\quad C = 100$





5 The 20Mf in RC Circuit shown in Fig. 2 has an initial charge $q = 500$ microcolomb, with polarity shown, at $t = 0$ the switch is closed thereby applying The Constant Voltage $V = 50$ volts, Find the Current transient.



كل قطب له ± إشارة

Sol.

S → closed

$$50 = 1000 i + \boxed{\frac{1}{C}\int i\,dt} \rightarrow$$

$$50 = 1000 i + \frac{1}{20 \times 10^{-6}}\int i\,dt$$

$$50 = 1000 i + 50000 \int i\,dt$$

بتفاضل الطرفين

$$0 = 1000 \frac{di}{dt} + 50000 i$$

$$0 = \frac{di}{dt} + 50 i \qquad \therefore (50 + D) i = 0$$

one solution only → P.I $\quad i = A e^{mt}$

$\boxed{m = -50}$

→ at $t = 0$ $\qquad i = 50/1000 +$ initial voltage ~~current due~~ to initial charge/R

→ initial voltage ~~current~~ due to charge $= q_0/c$ $\qquad \Rightarrow \boxed{q = CV}$

$$= 500 \times 10^{-6} / 20 \times 10^{-6} = 25V$$

لو افترضنا t = صفر فيه في موجود شحنة قيمتها $q_0/c = 25V$

وحتى تعطى الشحنة نفس اتجاه تيار القيم فيه ولذلك نجمع

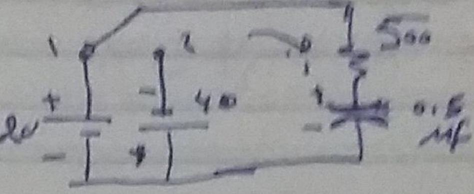
$$i_0 = (V + q_0/c)/R = (50+25)/1000 = 0.075A$$

→ at $t = 0$ $\qquad i = 0.075 = A e^{0} \qquad \therefore A = 0.075$

$$\therefore \boxed{i = 0.075 e^{-50t}}$$

6) in The RC circit shown in figure 3 The Switch is closed in Position 1 at $t=0$, after $1\,\tau$ moved to 2. Find complete Current Transient

Sol.



at Position 1

$$20 = 500\,i + \frac{1}{C}\int i\,dt$$

بالتفاضل : $0 = 500\frac{di}{dt} + \frac{1}{0.5\times10^{-6}}\,i$ $\quad (\; 0 = 500\,Di + 2000000\,i$

$$0 = Di + 4000\,i \qquad i(D+4000) = 0$$

$$\boxed{i_1 = A\,e^{-4000 t_1}}$$

at $t=0$ → $i = 20/500 = 0.04 = A$

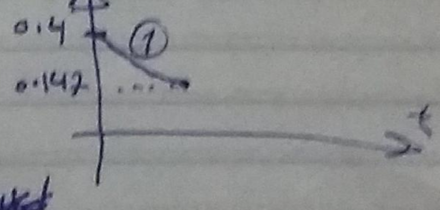
$$\therefore\ i_1 = 0.04\,e^{-4000 t_1}$$

يستمر هذا التيار لمدة $250\,\mu sec = (1RC) = 1\,TC$

after $1\,TC = 1RC = 250\,\mu sec$

$$i = 0.04 \times e^{-4000\times 250\mu} = 0.147\,A$$

Now switch moved to 2



$$-40 = 500\,i + \frac{1}{C}\int i\,dt$$

$$0 = 500\,di/dt + \frac{1}{0.5\times10^{-6}}\,i$$

$$i_2 = B\,e^{-4000(t-t_1)}$$ ~~initial~~

at $t = t_1 = 250\,\mu sec$ لاحظ أن زمنا معادلة الجهد في 2 لازال المكثف مشحون بالجهد الذي عليه من الـ 1 وهو

$$\frac{(0.04)}{0.5\times10^{-6}}\ \frac{e^{-4000t}}{-4000} + K = \frac{1}{C}\int i\,dt = V_c$$

$$V_c = -20\,e^{-4000t} + K \qquad \text{at } t=0 \quad K = +20 \quad \left(\begin{matrix} V_c = 0 \\ V_R = V \end{matrix}\right)$$

$$\therefore V_c = 20(1 - e^{-4000t}) \rightarrow \tau = RC = 1\,TC$$

ش $v_c = 20(1-e^{-4000t}) = 20(1-e^{-4000\times RC})$ ($RC = 250\times10^{-6}$)

لحظة زمن $= 1RC$

$= 20(1-e^{-1}) = 12.65$ Volt

∴ $\boxed{i_2 = B e^{-4000(t-t_1)}}$

لأن عند اتجاه قطبية C مع اتجاه قطبية البطارية 40V

at $t = t_1 \Rightarrow i = \frac{v_{total}}{R}$

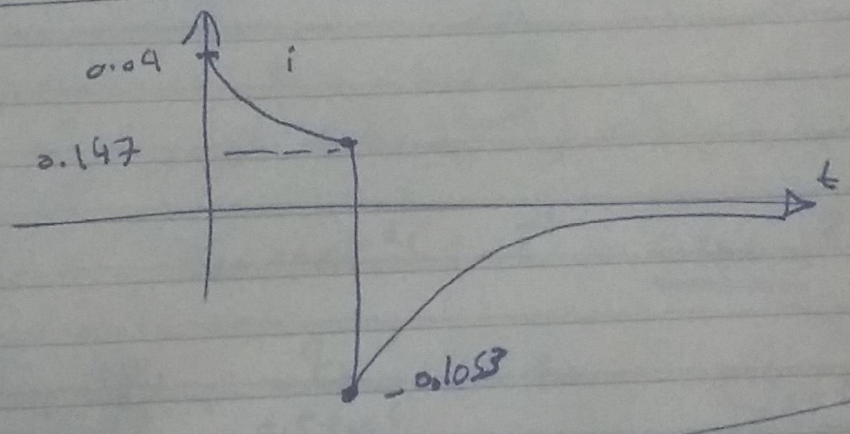
البطارية مكثف

$i = -\frac{40+12.65}{500} = -0.1053A$

لأن الاتجاه عكس ~~...~~ تيار البطارية عكس

اتجاه i المرسوم

∴ $\boxed{i_2 = -0.1053\, e^{-4000(t-t_1)}}$



توضيح :-

$-40 - v_{c_1} = 500 i + \frac{1}{C}\int i\,dt$

الجهد المتبقي من الدائرة السابقة — الجهد الجديد للدائرة 2

$1C = T$:

$v_{c_1} = \frac{1}{C}\int i_1\,dt = \square + K = 20(1-e^{-4000t}) = 12.65$ (→ 1C)

∴ $-40-12.65 = 500 i + \frac{1}{C}\int i\,dt$ الحل من الدرجة الأولى

$i_2 = B e^{-4000(t_2)}$ متغير

at $t_2 = 0$: $i_2 = B$ & المعادلة الأولى تصبح

$-40-12.65 = 500 i$

$i = -0.1053 = B$

∴ $i = -0.1053\, e^{-4000 t_2}$

[7] Determine The charge Transient for Prob. 6 and differentiate to obtain Current

at position 1

$$500 \frac{dq}{dt} + \frac{q}{0.5\times10^{-6}} = 20$$

$\therefore (D+4000)\,q = 0.04$

Sol

$q = Ae^{mt} + B$

$m = -4000$ $\quad B = \frac{0.04}{4000} = 10\times10^{-6}$

$\therefore \boxed{q_1 = Ae^{-4000t} + 10\times10^{-6}}$

at $t=0$, $q_0 = 0$ $\quad \therefore A = -10\times10^{-6}$

or $q_1 = 10\times10^{-6}\,(1 - e^{-4000t}) \longrightarrow (1)$

after $t = 1TC$ $\quad q_0 = 10\times10^{-6}(1-e^{-1}) = 6.32\times10^{-6}$

at S_2

$$500 \frac{dq_2}{dt} + \frac{q_2}{0.5\times10^{-6}} = -40 \qquad (t_2 = t - t_1)$$

$(D+4000)\,q = -0.08$

Solution $\quad q_2 = Ce^{m_2 t_2} + K$ $\qquad m_2 = -4000$

$K = \frac{-0.08}{4000} = -20\times10^{-6}$

$q_2 = Ce^{-4000t_2} - 20\times10^{-6}$

at $t_2 = 0$ $(t = t_1) \rightarrow q_2 = 6.32\times10^{-6}$ $\qquad \therefore \boxed{C = 26\times10^{-6}}$

$$\therefore \boxed{q_2 = 26\times10^{-6}\,e^{-4000t} - 20\times10^{-6}}$$

$t < t_1$

$$i_1 = \frac{d}{dt}\left\{10\times10^{-6}(1-e^{-4000t})\right\} = 0.04\,e^{-4000t}$$

$t > t_1$

$$i_2 = \frac{d}{dt}\left\{26.32\times10^{-6}\,e^{-4000(t-t_1)} - 20\times10^{-6}\right\}$$

$$= -0.153\,e^{-4000(t-t_1)}$$